T- Test
(Part 1/2)
(Parametric Test)

- ✓ Concepts
- ✓ Formulas for different Condition
- ✓ T-Test for Random Sample
- ✓ T-Test for Observation of Correlation

Biostatistics & Research Methodology
B Pharm 8th Sem | M. Pharm. | PhD

Hypothesis Testing

Parametric Test

- **Z Test**
  - N>30
  - Variance Known
  - Comparing the mean
- **T-Test**
  - N<30
  - Variance in unknown
  - Comparing the mean
- **F-Test**
  - Comparing the SD
- **ANOVA**
  - Comparing > 2 group
  - One Way, Two Way, Multiple Way

Non Parametric Test

- **Goodness of Fit**
  - X2 Test
  - Anderson darling
  - Kuiper's Test
  - Hosmer Lemeshow Test
- **Test of Independence**
  - X2 Test
  - Fisher's Exact
- **Test of Homogeneity**
  - X2 Test
  - Wilcoxon Rank test
  - Mann Whitney Test
  - Kruskal Wallis
  - Friedmann's Test
  - Levene Test

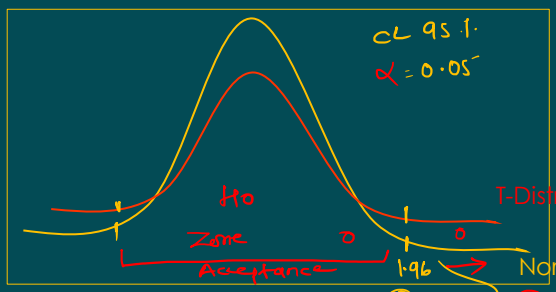
H_0 Null → $\bar{x} = \mu$, $\bar{x} = \bar{y}$
H_1/H_a Alternate → $\bar{x} \neq \bar{y}$, $\bar{x} \neq \mu$

H_0 accept $\bar{x} = \mu$
H_0 reject $\bar{x} \neq \mu$

T- Test

- Also Known as Student T-Test
- It is parametric test used when sample size ≤ 30 and Variance is unknown
- It Statistically compare between two means of groups/sample
- No. of Comparing mean- 2 (X vs Y or X vs μ)

N	Degree of Freedom	T (0.05)
5	4	2.77
10	9	2.26
30	29	2.04
>30	-	1.96 (Z 0.05)



T- Test

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

T- Test

- **Formulas for calculating t value:**

$n < 30$ $\quad \bar{x}$ Vs μ $\quad t = \frac{\text{Mean Diff}}{SEm}$

1. For mean of Random Sample

$$t = \frac{|\bar{x}-\mu|}{SEM} = \frac{|\bar{x}-\mu|}{s/\sqrt{n}} = \frac{|\bar{x}-\mu| \cdot \sqrt{n}}{s}$$

$$s\ (\text{Std. deviation}) = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}}$$

$\because$ $\bar{x}$ = mean of Sample
μ = mean of population

Conf. Interval of α level $\quad SEm = \frac{s}{\sqrt{n}}$

CI = mean $\pm$ t_{α} . SEm — one tail

CI = mean $\pm$ $t_{\alpha/2}$. SEm — two tail

drug A $\bar{x}$ Vs drug B $\bar{x}_1/\bar{x}_2$ — Unpaired

2. For mean of two independent Sample (Unpaired T-Test)

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s} \times \sqrt{\frac{n_1 \times n_2}{n_1 + n_2}}$$

$$s = \sqrt{\frac{\Sigma (x_1-\bar{x}_1)^2 + \Sigma (x_2-\bar{x}_2)^2}{n_1+n_2-2}}$$

T- Test

- **Formulas for calculating t value:**

3. For mean of two dependent Sample (Matched Paired T Test)

$$t = \frac{\bar{d} \times \sqrt{n}}{s}$$

$$s = \sqrt{\frac{\Sigma (d-\bar{d})^2}{n-1}}$$

$\because$ $\bar{d}$ = mean of difference

$n = 6$
$df = 5$

drug A

Before	After
1	
2	
3	
4	
5	
6	

4. For observed Correlation Coefficient

$$t = \frac{r}{\sqrt{1-r^2}} \times \sqrt{n-2}$$

$\because$ n = no. of Sample
r = Correlation Coefficient

QA = QB — (r)

T- Test

1. For mean of Random Sample

Q. A company claim that the average life time of a glucometer is 20 months, and the life of 5 samples of them is 21, 18, 17, 22, 17. so check that is company claim is right or not at 1 % of Significance level ($t_{0.01,4} = 4.60$)

$t = \frac{|\bar{x}-\mu|}{SEM} = \frac{|\bar{x}-\mu|}{s/\sqrt{n}} = \frac{|\bar{x}-\mu|\cdot\sqrt{n}}{s}$

s (std. deviation) $= \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$

$\therefore$ $\bar{x}$ = mean of Sample

μ = mean of population

$\mu = 20$

$\bar{x}$

SN	X	X-$\bar{X}$	$(X-\bar{X})^2$
1	21	2	4
2	18	-1	1
3	17	-2	4
4	22	3	9
5	17	-2	4
N = 5	ΣX^2 95/5		22
	$\bar{X}$ = 19		$\Sigma(X-\bar{X})^2$

① Assumption = $H_0 - \bar{x} = \mu$

$H_a \Rightarrow \bar{x} \neq \mu$

② Sig. level = 1% $t_{0.01,4} = 4.60$ t_{tab}

③ $t = \frac{|\bar{x}-\mu|}{s}\cdot\sqrt{n}$

$= \frac{|19-20|}{2.34}\cdot\sqrt{5} = \frac{1}{2.34}\cdot 2.23 = 0.95$

$s = \sqrt{\frac{22}{4}} = \sqrt{5.5} = 2.34$

$t_{cal} = 0.95$

H_0 H_1 t_{tab}

④ Conclusion = $t_{cal} < t_{tab}$ = H_0 Accepted

$= \bar{x} = \mu = 19 = 20$

$\therefore$ The claim is right

$\bar{X} = 19$

$n-1 = 4$

$\Sigma(X-\bar{x})^2 = 22$

T- Test

2. For observed correlation coefficient

$t = \frac{r}{\sqrt{1-r^2}}\cdot\sqrt{n-2}$

Q. A random sample of 27 observations give a correlation coefficient r= 0.5, is it likely the variables in the population are corelated ?

① Assumption → H_0 → Variables are correlated

H_a → Variables are uncorrelated

② Significance level – 5% , $t_{0.05,25} = 2.06$ df = n-2

③ Calculation – $t = \frac{r}{\sqrt{1-r^2}}\cdot\sqrt{n-2}$

$= \frac{0.5}{\sqrt{1-(0.5)^2}}\cdot\sqrt{27-2}$

$= \frac{0.5}{\sqrt{1-0.25}}\cdot\sqrt{25}$

$= \frac{0.5}{\sqrt{0.75}}\cdot 5 = \frac{2.5}{0.86} = 2.90$

t_{cal} 2.9

④ Conclusion

$t_{cal} > t_{tab}$

(2.9) (2.06)

↳ H_0 reject

↳ H_a accept = differ

* Variables in the population are uncorrelated.

T- Test
(Part 2/2)
(Parametric Test)

Paired and Unpaired T Test

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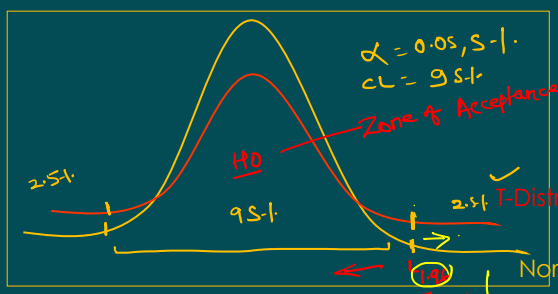

T- Test

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df											
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4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
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7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
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13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

T- Test

- **Formulas for calculating t value:**

$\bar{x}$ Vs μ

1. For mean of Random Sample

$$t = \frac{|\bar{x}-\mu|}{SEM} = \frac{|\bar{x}-\mu|}{s/\sqrt{n}} = \frac{|\bar{x}-\mu| \cdot \sqrt{n}}{s}$$

$$s\ (\text{std. deviation}) = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n-1}}$$

$\therefore$ $\bar{x}$ = mean of Sample

μ = mean of population

Conf. Interval of α level

CI = mean $\pm$ t_{α} . SEM — one tail

CI = mean $\pm$ $t_{\alpha/2}$. SEM — two tail

2. For mean of two independent Sample (Unpaired T-Test)

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s} * \sqrt{\frac{n_1 * n_2}{n_1 + n_2}}$$

$$s = \sqrt{\frac{\Sigma (x_1-\bar{x}_1)^2 + \Sigma (x_2-\bar{x}_2)^2}{n_1+n_2-2}}$$

T- Test

Formulas for calculating t value:

3. For mean of two dependent Sample (Matched Paired T Test)

$$t = \frac{\bar{d} \times \sqrt{n}}{s}$$

$$s = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}}$$

$\because \bar{d}$ = mean of difference

4. For observed Correlation Coefficient

$$t = \frac{r}{\sqrt{1-r^2}} \times \sqrt{n-2}$$

$\because n$ = no. of Sample
r = Correlation Coefficient

T- Test

3. Paired T-Test

Q. Check the test drug A is effective or not by following Data at 95% 0f confidence level ($t_{0.05, 4} = 2.77$)
Blood Suger Level in mg/dL on -
X (before drug A): 110, 120, 115, 120, 120
Y (After drug A): 90, 110, 80, 100, 110

$n = 5$

$t = \frac{\text{diff. in mean}}{\text{sem}}$, $\text{sem} = \frac{SD}{\sqrt{n}}$

① Assumption → $H_0 \rightarrow \bar{x} = \bar{y}$ OR $H_1 \rightarrow \bar{x} \neq \bar{y}$

② Sig. level → 5%; $t_{0.05, 4} \rightarrow 2.77$ (table)

③ Calculation :- $t = \frac{\bar{d} * \sqrt{n}}{s}$, $s = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}}$ (SD)

- $d = y - x$ or $x - y$
- $\bar{d} = \Sigma d / n$
- $d - \bar{d} = ?$
- $(d-\bar{d})^2 = ?$

T- Test

3. Paired T-Test

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Blood Suger Level in mg/dL on -
X (before drug A): 110, 120, 115, 120, 120
Y (After drug A): 90, 110, 80, 100, 110

X	Y	d (X-Y)	d-$\bar{d}$	(d-$\bar{d}$)2
110	90	20	1	1
120	110	10	-9	81
115	80	35	16	256
120	100	20	1	1
120	110	10	-9	81
		95/5 = $\bar{d}$ 19	$\Sigma(d-\bar{d})^2$	420

③ Calculate
↳ $n = 5$
↳ $\bar{d} = 19$
↳ $\Sigma(d-\bar{d})^2 = 420$

$t = \frac{\bar{d} \times \sqrt{n}}{s}$, $s = \sqrt{\frac{\Sigma(d-\bar{d})^2}{n-1}} = \sqrt{\frac{420}{4}} = \sqrt{105} = 10.24$

$= \frac{19 \times \sqrt{5}}{10.24}$

$= \frac{19 \times 2.23}{10.24} = \frac{42.37}{10.24} = 4.13$

t_{cal} 4.13 > 2.77 H_0 rejected

④ Conclusion $t_{cal} > t_{tab}$, H_0 rejected, H_1 accepted
4.13 2.776
$\bar{x} \neq \bar{y}$
↳ drug A is effective

T- Test

4. Unaired T-Test

Q. Check the test drug A and drug B has same effective or not by following Data at 95% 0f confidence level (t0.05, 8 = 2.306)
Body weight reduction Level in Kg on -
X1 (drug A): 8, 10, 12, 10, 10 $n_1 = 5$
X2 (drug B): 4, 4, 3, 2, 2 $n_2 = 5$

① Assumption → H_0 $\bar{x}_1 = \bar{x}_2$, Same effective
H_a $\bar{x}_1 \neq \bar{x}_2$, has different effect-

② sig. level → 5%, $t_{0.05,8} = 2.306$ ↳ t_{tab}

③ calculation → $t = \frac{|\bar{x}_1 - \bar{x}_2|}{s} * \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$

$s_p = \sqrt{\frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1 + n_2 - 2}}$

T- Test

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Body weight reduction Level in Kg on -
X1 (drug A): 8, 10, 12, 10, 10
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① Assumption → H_0 $\bar{x}_1 = \bar{x}_2$, Same effective
H_a $\bar{x}_1 \neq \bar{x}_2$, has different effect-

② sig. level → 5%, $t_{0.05,8} = 2.306$ ↳ t_{tab}

③ calculation → $$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s} \cdot \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$s = \sqrt{\frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}}$$

T- Test

4. Unaired T-Test

Significantly ($P < 0.05$)

Q. Check the test drug A and drug B has same effective or not by following Data at 95% 0f confidence level (t0.05, 8 = 2.306)
Body weight reduction Level in Kg on -
X1 (drug A): 8, 10, 12, 10, 10
X2 (drug B): 4, 4, 3, 2, 2

	X1	X1-$\bar{X}$1	(X1-$\bar{X}$1)^2	X2	X2-$\bar{X}$2	(X2-$\bar{X}$2)^2
1	8	-2	4	4	1	1
2	10	0	0	4	1	1
3	12	2	4	3	0	0
4	10	0	0	2	-1	1
5	10	0	0	2	-1	1
Sum-	50		8	15		4

$\bar{x}_1 = 50/5 = 10$ $\bar{x}_2 = 15/5 = 3$

③ calculation $t = \frac{|\bar{x}_1 - \bar{x}_2|}{s} \cdot \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$

$s = \sqrt{\frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}}$

$= \sqrt{\frac{8+4}{5+5-2}} = \sqrt{\frac{12}{8}} = \sqrt{1.5} = 1.22$ (SD)

$t = \frac{|10-3|}{1.22} \cdot \sqrt{\frac{5 \times 5}{5+5}} = \frac{7}{1.22} \cdot \sqrt{\frac{25}{10}}$

$= \frac{7}{1.22} \cdot \sqrt{2.5} = \frac{7}{1.22} \cdot 1.58 = \frac{11.06}{1.22} = 9$

$t_{cal} = 9$

④ Conclusion → $t_{cal}(9) > t_{tab}(2.30)$
→ H_0 rejected $\bar{x}_1 \neq \bar{x}_2$, $\bar{x}_1 > \bar{x}_2$
→ drug A is more effective than B